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INFORMATION TECHNOLOGIES UNDER THE MANIFOLD COORDINATE SYSTEMS[1]

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Symmetry, as wide or as narrow as you may define its meaning, is one which man through the ages has tried to comprehend and create order, beauty and perfection.

H. Weyl

INTRODUCTION

This work presents information technologies under manifold coordinate systems for improving the quality indices of the technologies and management systems with respect to vector data coding and compression, transmission speed, and embody reliability, using design based on the combinatorial properties of rotational symmetry and complementary asymmetry relationship. The topological model of the coordinate systems regarded as both algebraic constructions, based on cyclic groups in extensions of Galois fields, and intelligent non-redundant combinatorial configurations, generated from "elegant" ensembles of rotational symmetry composed from complementary asymmetries. These design techniques make it possible to configure information technology with vector data indexing and processing under basis of two- and multi-dimensional coordinate system, where basis is a sub-set of general number indexed vector data "category-attribute", which belong to mapping nodal coordinate points set of the system. The basis generates indexed vector data "category-attribute" set using modular summing for complete a

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reference grid of the coordinate system. Moreover, we require each indexed vector data "category-attribute" mutually uniquely corresponds to the point with the eponymous set of the coordinate system. In practice, the set points obtained using optimized basis of the system. This methodology working out harmonious mutual penetration of rotational symmetry and asymmetry as the remarkable property of real space for configure multi-attribute intelligent information management technologies under the coordinate system. Besides, a combination of binary code with vector weight bits of the database allowed, and the set of all values of indexed vector data sets one-to-one correspondents to nodal points set of the *t*-dimensional coordinate system. The underlying mathematical principle relates to the optimal placement of structural elements in spatially or temporally distributed systems, using novel designs based on *t*-dimensional combinatorial configurations, including the appropriate algebraic theory of cyclic groups, number theory, modular arithmetic, and geometric transformations. This information technology brought out relationship between the "elegant" ensembles of rotational symmetry and intelligent models of toroidal coordinate systems.

1. The "elegant" symmetry and asymmetry completed system

Symmetry and asymmetry relation in geometric construction is the most familiar type of the real world shapes. In mathematics, the more general meaning of symmetry-asymmetry have combinatorial configurations. The next viewpoint describes rotational symmetry as a set of two subsets embedded into the symmetry. In this context, symmetries and asymmetries underlie some of the most profound results relating found in systems engineering, including aspects of modern information technologies. Finally, symmetry-asymmetry ensembles are features that can described as exhibiting manifold forms of spatial harmony and "perfect" relationships encoded into rotational symmetry of higher dimensionality. The role of such relationships is widespread applicability, and has been extremely effective when applied in simplifying solutions to many problems for finding the optimum ordered arrangement of structural elements in distributed technological systems.

Two aspects of the matter the issue are examined useful in applications of symmetrical and non-symmetrical models: optimization of technology, and hypothetic unified "universal informative field of harmony"[2].

[2] Wigner, E. (1971). *Sketches on symmetry*. (Trans. From English). Moscow : Mir, 320 p.

Let us regard *S*-fold rotational symmetry as an ability to reproduce the maximum number of combinatorial varieties in an asymmetry, embedded into the *S*-fold symmetry, using its two-part division over a space relative to central point of the symmetry.

To extract meaningful information from the underlying equation let us apply to rotational *S*-fold symmetry as a planar space of two complementary completions of the symmetric space.

For example, the 3-fold (S=3) rotational symmetry incorporates two complementary asymmetries: n_1 =1, and n_2 =2 (Fig. 1).

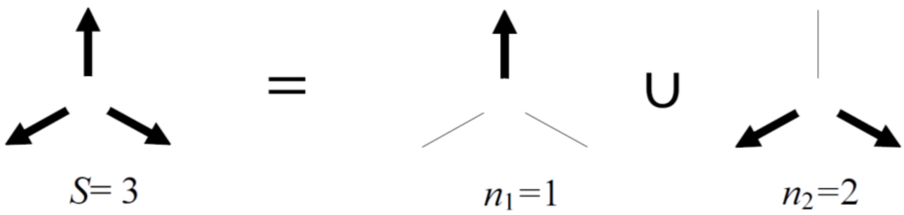

Fig. 1. The 3-fold (*S*=3) rotational symmetry incorporates two complementary asymmetries: n_1 =1, and n_2 =2

Regarding $n_1 = 1$ and $n_2 = 2$ lines as being complementary sets of 3-fold symmetry, where $S = n_1 + n_2$, we require a set of all angular distances enumerates the set of spacing angles [α_{min}, 360º– α_{min}] fixed number
R-times for each of the sets. Here the set of spacing angles for n_1 =1 is exactly R_1=1 time of finite interval [360º, 0º], while R_2 =1 for n_2 =2 of interval [120º, 240º].

Regarding n_1 =1 and n_2=2 arrows as being complementary sets of 3-fold symmetry, we require a set of all angular distances enumerates the set of spacing angles [α_{min}, 360º– α_{min}] fixed number *R* -times for each of the sets. Here the set of spacing angles for n_1 =1 is exactly R_1=1 time of finite interval [360º, 0º], while R_2 =1 for n_2 =2 of interval [120º, 240º], $S = n_1 + n_2$.

Our reasoning proceeds from the fact, that the minimal and maximal angular distances relation initiated by *S*-fold rotational symmetry to be of prime importance for discovery of the *S* –fold “elegant” symmetry-asymmetry ensemble (Fig. 2).

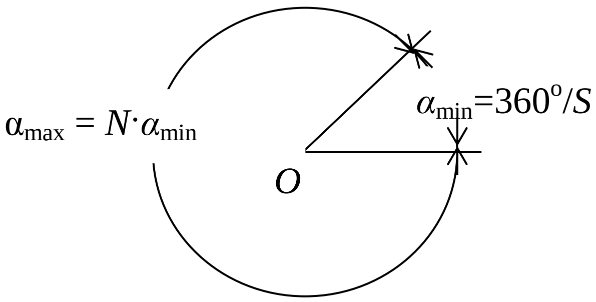

Fig. 2. A chart for discovery of the *S* –fold "elegant" symmetry-asymmetry ensemble

We require the set of all N angular distances [α_{min}, $N - \alpha_{min}$] of S-fold quantized space divided by a set of n straight lines diverged from a central point O non-uniformly allows an enumeration of all integers [1, S–1] exactly R-times. If these requires request, we call this phenomenon the "perfect" rotational S-fold symmetry. From Fig. 2 follows integer relation between of variables S, n, and R:

$$S = \frac{n(n-1)}{R} + 1 \tag{1}$$

As follows from equation (1), there are exist an infinite number of the "perfect" ensembles, and this is a necessary, but not a sufficient condition.

The elementary model of a non-redundant dial follows from 3-fold rotational symmetry (S=3) that splits into 1-fold (n_1=1, R_1=1) and 2-fold (n_2=2, R_2=1) asymmetric components (Fig. 1). To see this, we observe that minimal angular interval between lines (α_{min}) of the rotational symmetry is equal to $\alpha_{max} = 360^o/S = 360^o/3 = 120^o$. The first asymmetric component enumerates the set {1} by step $\alpha_{min}=360^o$ exactly once (R_1=1), and the second- the set of integers {1, 2} by step $\alpha_{min}=120^o$ exactly once (R_2=1) also.

Ensemble of the "perfect" rotational symmetry-asymmetry of order 7 (S= 7) shows schematically below (Fig. 3).

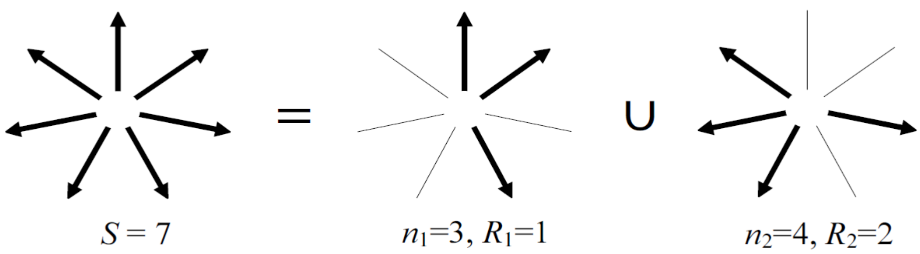

Fig. 3. Ensemble of the "perfect" rotational symmetry-asymmetry of order 7 (*S*= 7)

The 7-fold rotational symmetry (S=7) splits into 3-fold (n_1=3) asymmetry, which allows an enumeration the set of all angular intervals [360º/7, 6×360º/7] of n_1=3, R_1=1, and n_2 =4, R_2 =2 by quantization level α_{min}=360º/7 (Fig. 3). This rotational symmetry allows an enumeration the set of all angular intervals between three (n_1=3) heavy faced lines from α_{min}=360º/7 to n_1 (n_1-1) α_{min}= 6α_{min} exactly once (R_1=1), while set of all intervals between four (n_2=4) thin faced ones – exactly twice (R_2=2). Phenomenon of the "perfect" rotational S-fold symmetry splits into odd and even asymmetric complementary components (heavy and thin faced lines), and each of them enumerates the set of integers [1, S-1] the precise numbers of times. Note, the behaviour of the phenomenon displays is odd S-fold symmetry, while even S-fold rotational symmetry occurs very seldom in this role.

Numerical data $7 \leq S \leq 19$, $n_{1,2} \geq R_{1,2}$ +2 for analysis of the "perfect" rotational S-fold symmetry are tabulated (Table 1).

Table 1

Numerical data 7 ≤ *S* ≤ 19, and $n_{1,2} \geq R_{1,2}$ +2

№	$S= n_1+n_2$	n_1	R_1	n_2	R_2
1	7	3	1	4	2
2	11	5	2	6	3
3	13	9	6	4	1
4	15	7	3	8	4
5	19	9	4	10	5

From Table 1 we can see that the rotational symmetry-asymmetry phenomenon is two complementary asymmetries of even (n_1), and odd (n_2) orders, each of them allows an enumeration the set of angular distances exactly R-times.

The symmetry and asymmetry relationships discover fundamental property of the real space-time. It can be extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in spatially or temporally distributed technological and information systems. Research into the underlying problem relating to the profitable placement of elements in the system with ring structure generalizes pattern numerical model, namely the concept of Ideal Ring Bundles (IRBs)[3].

[3] Riznyk, V. (2017). Multi-modular Optimum Coding Systems Based on Remarkable Geometric Properties of Space. *Advances in Intelligent Systems and Computing*, 512, 129-148. https://doi.org/10.1007/978-3-319-45991-2_9

An one-dimensional IRB is cyclic sequence of positive integers which form "perfect" partitions of a finite interval [1, S] of integers. The sums of connected sub-sequences of an IRB enumerate the set of integers exactly R-times.

To continue we refer to the properties inherent in the relation based on phenomenon of "perfect" rotational symmetry-asymmetry relationship.

Study of the "perfect" relationship includes use of modern mathematical methods of optimization of systems that exist in the theory of combinatorial configurations[4], and algebraic number theory. One-dimensional (1-modular) optimum relationships are cyclic sequences of positive integers. An n-stage ring sequence $C_n = \{k_1, k_2, \ldots, k_n\}$ of positive integers for which the set of all sums of connected sub-sequences of the sequence enumerate the set of integers [1, S–1] exactly R-times. For example, cyclic sequence {1, 2, 6, 4} (mod 13) is IRB containing four (n=4) elements allows an enumeration of all numbers: 1=1, 2=2, 3=1+2, 4=4, 5=4+1, 6=6, 7=4+1+2, …, 12=2+6+4 exactly once (R=1). If this sequence is converted to the 1 : 2 : 6 : 4 circular ratio we call this 1D "optimum 1-modular relationship" of order n=4, R=1, and S=13.

For any prime power q there exists a finite field GF(q)[5].

The multiplicative group of GF(q) is cyclic; thus it is generated by any of its $\varphi(q-1)$ elements of order q–1. These generating elements are primitive roots and for prime p, the residues 0,1,…, p–1 form a field with respect to addition and multiplication modulo p. GF(q^m) is represented by the set of all m-tuples with entries from GF(q). In this representation addition is performed component wise but multiplication is more complicated. Associate with the m-tuple a_{m-1}, a_{m-2}, …, a_1, a_0 the polynomial $a_{m-1} x^{m-1} + \ldots + a_1 x + a_0$. Then, in order to multiply two m-tuples, multiply instead their associated polynomials and reduce the result modulo any fixed m^{th} degree polynomial $f(x)$ irreducible over GF(q). The coefficients of the resulting polynomials constitute the m- tuple, which is the product of the original two. For multiplicative purposes it is more convenient to represent GF(q^m) in terms of a primitive root α; in which case, GF(q^m) consists of 0, $\alpha^0, \alpha^1, \alpha^2, \alpha^3, \ldots, \alpha^{r-2}$ where $r = q^m$.

This represents the set of elements for finite projective geometry PG (2, 3), of dimension N=2 over GF(3), with $f(x) = x^3 - x - 1$:

[4] Hall, M. (1970). *Combinatorics*. (Trans. from English). Moscow: Mir, 424 p.

[5] Rotman, J. (1998). *Galois Theory*. SpringerLink, 106 p. https://doi.org/10.1007/978-1-4612-0617-0

$$\left.\begin{aligned}
x^0 &\equiv 1\\
x^1 &\equiv x\\
x^2 &\equiv x^2\\
x^3 &\equiv x+1\\
x^4 &\equiv x^2+x\\
x^5 &\equiv x^2+x+1\\
x^6 &\equiv x^2+2x+1\\
x^7 &\equiv 2x^2+2x+1\\
x^8 &\equiv 2x^2+2\\
x^9 &\equiv x+2\\
x^{10} &\equiv x^2+2x\\
x^{11} &\equiv 2x^2+x+1\\
x^{12} &\equiv x^2+2
\end{aligned}\right\}\ (\text{mod}d\ 3, x^3-x-1)$$

Easy to see, that $x^i \leftrightarrow i$, $i = 0,1,\ldots, S_n - 1 = n(n-1) = 12$, where $n = 4$. Here sequence of all fixed elements of zero coefficients by x^2 are as follows: 1, x, x+1, x+2. We regard the PG(2,3) as a central symmetrical figure {0, 1, …,12} of order $S_=13$, where elements 1, x, x+1, x+2 generate cyclic proportion 1:2:6:4 (Fig. 4). This proportion is optimum 1-modular (mod 13) relationship of order four (n=4), where k_1=1, k_2=2, k_3=6, k_4=4.

The elements of the Galois field arranged in 13 (S =13) tops of the circular graph (Fig. 4).

Fig. 4 displays two non-uniform polygons of four (n_1=4), and nine (n_2=9) vertexes embedded in 13-fold rotational symmetry. The first of them places in the vertexes x^0, x^1, x^3, x^9 is the IRB {1, 2, 6, 4} with parameters S=13, n_1=4, R_1=1, and the second is the IRB {2,1,1,1,1,2,1,1,3} with S=13, n_2=9, R_2=6.

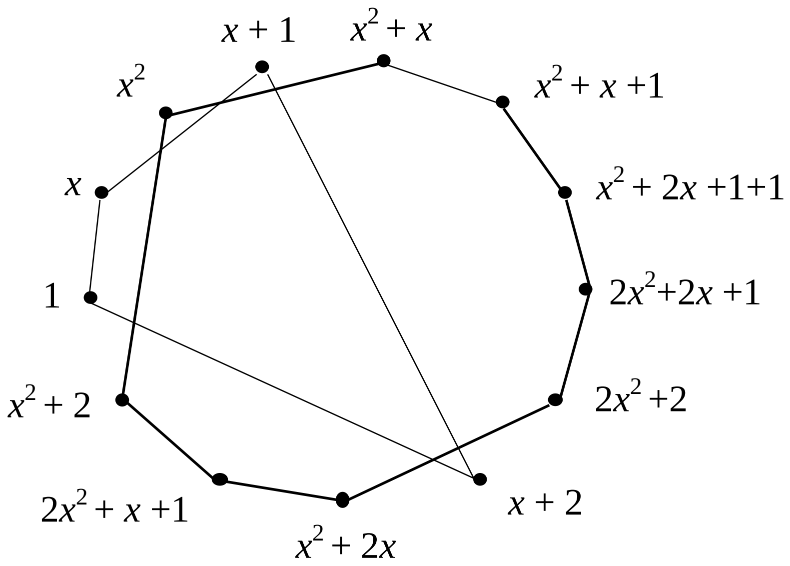

Fig. 4. Graphic representation of complementary IRBs with parameters n_1=4, S_n=13, R=1, and n_2=9, S=13, R=6 in terms of Galois field by $f(x)= x^3 – x – 1$

Next we regard n-stage ring sequence K_{2D}= $\{(k_{11}, k_{12}), (k_{21}, k_{22}),..., (k_{i1}, k_{i2}),...,(k_{n1}, k_{n2})\}$, where we require all terms in each circular vector-sum to be consecutive 2-stage (t=2) sequences as elements of the sequence. A circular vector-sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to n-1. An n-stage ring sequence K_{2D}, for which the set of all two-modular vector-sums (mod m_1, mod m_2) forms two-dimensional grid $m_1 \times m_2$ over a surface of a torus, where each node of the grid occurs exactly R-times, is named two-dimensional (t=2) Ideal Ring Bundle (2D IRB) with parameters n, R, and m_1, m_2.

Here are four variants of 2D IRBs with S =7, n = 3, R=1:

(a) {(1,0),(1,1),(1,2)}; (b) {(0,1),(0,2),(1,0)};

(c) {(0,1),(0,2),(1,2)}; (d) {(0,1),(0,2),(1,1)}

It known the fact that linear algebraic operations such as addition and multiplication can be consistently defined using finite sets of integers, by using modular arithmetic.

For example, variant (a) of the 2D IRB {(1,0), (1,1), (1,2)} gives the next vector sums modulo m_1 =2, m_2=3:

$$\left.\begin{array}{l}(1,0)+(1,1) \equiv (0,1)\\(1,1)+(1,2) \equiv (0,0)\\(1,2)+(1,0) \equiv (0,2)\end{array}\right\}(\operatorname{mod} 2, \operatorname{mod} 3)$$

So long as the vectors (1,0), (1,1), (1,2) of the 2D IRB themselves are two-dimensional vector sums also, the set of the modular ($m_1 = 2$, $m_2 = 3$) vector sums forms a set of two-modular reference grid over torus $m_1 \times m_2 = 2 \times 3$ exactly once

(R=1):	(0,0)	(0,1)	(0,2)
	(1,0)	(1,1)	(1,2)

Multiply the vectors {(1,0),(1,1),(1,2)} trough by coefficient (1,2) taking (mod 2), and (mod 3) as follows:

$$\left.\begin{array}{l}(1,0)\cdot(1,2) \equiv (1,0) \\ (1,1)\cdot(1,2) \equiv (1,2) \\ (1,2)\cdot(1,2) \equiv (1,1)\end{array}\right\}(\bmod 2, \bmod 3)$$

As a result of this transformation we got 2D IRB {(1,0),(1,2),(1,1)}, where vectors are in a reverse direction from vectors in the previous sequence, and the reverse transform by this multiplicative coefficient is true. Taking the same conversion for variants (b), (c), and (d), we obtain finally the next result: (a) × (1,2) ⇒ (a); (b) × (1,2) ⇒ (b); (c) × (1,2) ⇒ (d); (d) × (1,2) ⇒ (c).

Hence, four 2D IRBs {(a),(b),(c),(d)} form two isomorphic (a) and (b), and two non-isomorphic (c) and (d) variants of the 2D IRB. We call this the cyclic 2D IRB group. Note, each of these variants makes it possible to obtain exactly six ($m_1 \cdot m_2 = 6$) varied 2D vectors (numerical pairs) using only three (n=3) basic vectors.

The theoretical connection between the "elegant" symmetry, the Galois fields, and IRBs offer the great opportunities for development of advanced information technologies under manifold coordinate systems[6].

2. Manifold coordinate systems for innovative vector data coding

Development of the idea of "perfect" cyclic relationships embedded in the "elegant" symmetry comes to concept of manifold coordinate systems for innovative vector data coding.

Let us regard a graph of the 2D IRB {(0,1), (1,3), (0,2), (2,3)} with parameters n=4, R=1, m_1=3, m_2=4 depicted in a graph (Fig. 5).

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[6] Mathematics (2020). *Coordinate systems on manifolds*. Retrieved from: https://math.stackexchange.com/questions/1319939/coordinate-systems-on-manifolds

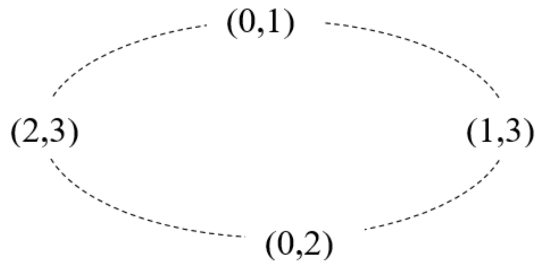

Fig. 5. A graph of the 2D IRB {(0,1), (1,3), (0,2), (2,3)} with parameters *n*=4, *R*=1, m_1=3, m_2=4

We can calculate easy all two-dimensional vector-sums, taking complex modulo (m_1 =3, m_2 =4):

(1,0)≡(0,1)+(1,3);	(1,1)≡(1,3)+(0,2);
(2,1)≡(0,2)+(2,3);	(2,0)≡(2,3)+(0,1);
(1,2)≡(0,1)+(1,3)+(0,2);	(0,0)≡(1,3)+(0,2)+(2,3);
(2,2)≡(0,2)+(2,3)+(0,1);	(0,3)≡(2,3)+(0,1)+(1,3).

So long as the vectors (0,1), (1,3), (0,2), (2,3) of the ring sequence themselves are circular 2-D vector-sums too, the complete set of these vector sums is as follows:

(0,0)	(0,1)	(0,2)	(0,3)
(1,0)	(1,1)	(1,2)	(1,3)
(2,0)	(2,1)	(2,2)	(2,3)

The result of the calculation is two-dimensional grid 3 × 4 over torus, where 2D coordinates of each node of the grid occurs exactly once (*R*=1). Therefore the ring sequence of the 2D vectors {(0,1), (1,3), (0,2), (2,3)} is 2D IRB with *n*=4, *R*=1, m_1=3, m_2=4.

Here is ring vector diagram of the 2D IRB {(0,1), (1,3), (0,2), (2,3)} (Fig. 6).

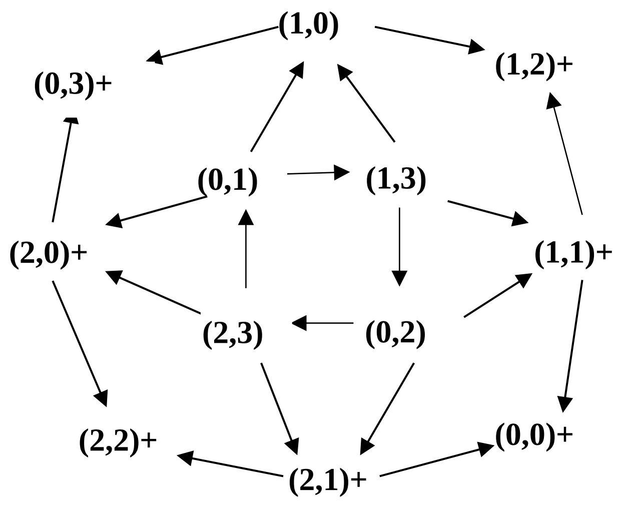

Fig. 6. A ring vector diagram of the 2D IRB {(0,1), (1,3), (0,2), (2,3)}

Diagram (Fig. 6) consists of four (n=4) n-stage sequences of 2D vectors, placed one inside another. The inner sequence is the 2D IRB {(0,1), (1,3), (0,2), (2,3)}, which forms the rest 2D sequences: {(1,0),(1,1),(2,1),(2,0)}, and finally {(1,2), (0,0), (2,2), (0,3)} taking summation with respect to modulo m_1=3 and m_2=4. The arrows point direction of the modular summing. We can observe that each modular sum from (0,0) to (2,3) occurs exactly once (R=1) on the diagram.

Generally, arbitrary 2D IRB with parameters S, n, R, m_1, m_2 make it possible to form a coordinate grid $m_1 \times m_2$ in common reference point (0,0) over surface of usual torus with two (t=2) concurrent axes (Fig. 7).

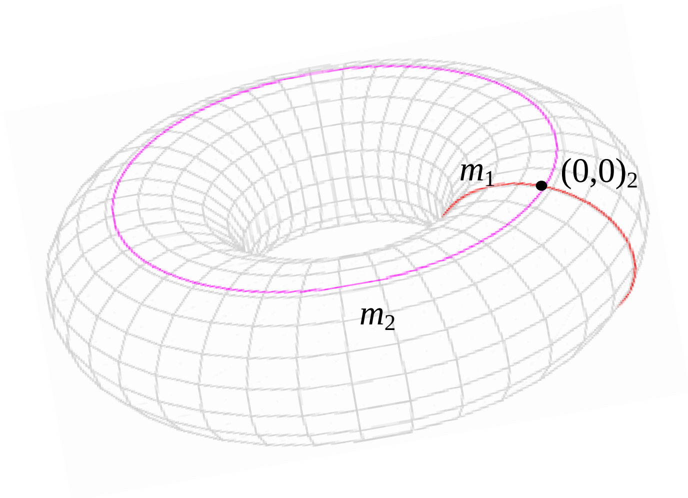

Fig. 7. A coordinate grid $m_1\times m_2$ in common reference point (0,0) over surface of usual torus with two (t=2) concurrent axes

Let calculate all S sums of the terms in the n-stage sequence of three-dimensional (t=3) sub-sequences of the sequence $K_{n3} = \{(k_{11}, k_{12}, k_{13}), (k_{21}, k_{22}, k_{23}),\ldots, (k_{i1}, k_{i2}, k_{i3}),\ldots, (k_{n1}, k_{n2}, k_{n3})\}$ as being cyclic. We require all terms in each 3-modular vector sum to be consecutive elements of the sequence, and a modulo sums m_1 of k_{i2}, m_2 of k_{i2}, and m_3 of k_{i3} are taken, respectively.

Here is an example of three-dimensional (t=3) vector ring n-sequence based on 3D IRB with n= 6, m_1 =2, m_2 =3, m_3 =5, and R=1 completed from six 3-stage (t=3) sub-sequences $\{K_1, K_2, \ldots, K_6\}$, where K_1 = (0,2,3), K_2 = (1,1,2), K_3 = (0,2,2), K_4 = (1,0,3), K_5 = (1,1,1), K_6 = (0,1,0). Here are all sums over the 6-stage sequence, taking 3- modulo (mod 2, mod 3, mod 5): (0,0,0) ≡ ((0,2,3) + (1,1,2) + (0,2,2) + (1,0,3) + (0,1,0)); (0,0,1) ≡ ((0,2,2) + (1,0,3) + (1,1,1)); (0,0,2) ≡ ((1,1,2) + (0,2,2) + (1,0,3)); …, finally, (1,2,4) ≡ ((0,2,3) + (1,1,2) + (1,1,1) + (1,0,3) + (0,1,0)).

The result of the calculation forms $m_1 \times m_2 \times m_3 = 2\times3\times5$ manifold grid, embracing a manifold surface enumerating nodal points shaped three-dimensional (t=3) coordinate system from (0,0,0) to (1,2,4) exactly once (R=1).

A t-manifold coordinate system immersed in (t+1) – dimensional no real space without self-intersection of coordinate axes. An t-dimensional coordinate system ($t > 2$) with t axes named the manifold coordinate system $m_1\times m_2\times\ldots\times m_t$. To discuss concept of perfect manifold coordinate system let us regard structural model of t-dimensional IRB as n-sequence $\{K_1, K_2, \ldots, K_i, \ldots, K_n\}$, of t-stage sub-sequences $K_i= (k_{i1}, k_{i2},\ldots, k_{it})$, $i = 1,2,\ldots,n$, each of them to be completed from t nonnegative integers, namely t-IRB with parameters S, n, R, t, m_i (i = 1, 2, …, t).

We refer again to the “perfect” t-stage (t-dimensional) n-sequence $\{K_1, K_2, \ldots, K_i, \ldots, K_n\}$ of t-stage sub-sequences of the sequence, each of them to be completed with nonnegative integers. The principal property of forming reference grid $m_1 \times m_2 \times\ldots\times m_t$ over a t-manifold surface is n-sequence of t-stage sub-sequences of the sequence, a set modulo sums taking t- modulo ($m_1, m_2, \ldots., m_t$) allows enumerating all coordinates of the t- manifold surface exactly R-times. We call this perfect t-manifold coordinate system $m_1 \times m_2 \times\ldots\times m_t$ with information parameters S, n, R, t, m_i (i = 1, 2, …, t). It is t-dimensional image surface involved spatially disjointed reference t-axes with common point.

A planar projection of t-dimensional manifold coordinate axes $m_1, m_2, \ldots, m_t$ for grid $m_1\times m_2\times\ldots\times m_t$ with common point “+” shows in Fig. 8.

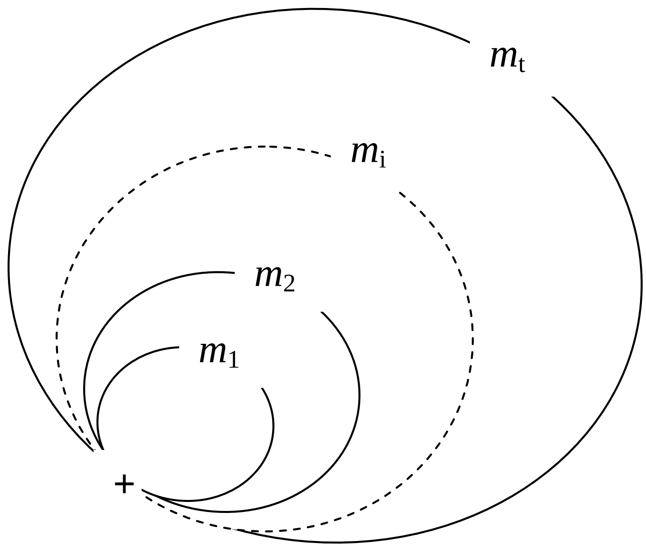

Fig. 8. A planar projection of *t*-dimensional manifold coordinate axes $m_1, m_2, \ldots, m_t$ for grid $m_1\times m_2\times\ldots\times m_t$ with common point “+”

The Fig. 8 demonstrates planar projection of spatially disjointed axes m_1, m_2, …, m_t of t-dimensional manifold reference grid $m_1 \times m_2 \times \ldots \times m_t$. The projection illustrates spatially disjointed axes m_1, m_2, …, m_t of manifold grid, which regains its multidimensional shape of a virtual surface.

Hence, in each case, the t--stage n- sequence forms manifold t-dimensional coordinate system as an object in dimension t+1, and a set of all t-modular sums taking mod m_1, mod m_2, …, mod m_t of the sequence enumerates nodal points of the coordinate grid with sizes $m_1 \times m_2 \times \ldots \times m_t$.

Clearly, an t-dimensional perfect manifold coordinate system designed for vector data indexing t attributes and m_i categories of each of them (i= 1,2,…, t) requires t concurrent disjointed axes m_1, m_2,…,m_i, …,m_t with common reference point for forming t- dimensional coordinate grid of the system with sizes $m_1 \times m_2 \times \ldots \times m_t$. The underlying coordinate system can be described by t-IRB with parameters S, n, t, m_i (i = 1, 2, …, t). Here S is an order of spatial symmetry, generated t-dimensional IRB, n- number of t-stage sub-sequences of n-sequence, and number of basic attribute-categories subset forming complete set of vector data array under the manifold coordinate system. In turn, m_i defines a number of categories of i- th attribute, as well as a number of reference points on i- th ring axis in a manifold coordinate system. Therefore, all information about t-dimensional vector data array of sizes $m_1 \times m_2 \times \ldots \times m_t$ embedded into the coordinate system.

The remarkable properties and structural perfection of t-IRBs allows configure high performance t-dimensional coding systems. The t-dimensional toroidal code combinations forms from t-stage n-sequences of an t-IRB. The t-modular sums of t-stage n-sequences of an t-IRB enumerate set of code combinations according to nodal points of the coordinate grid with sizes $m_1 \times m_2 \times \ldots \times m_t$. We call this “perfect” manifold code system.

For example, two-dimensional (t=2) IRB {(0,1),(1,0),(2,0)} forms binary perfect manifold 2D code under toroidal reference grid 3×2 (Table 2).

Table 2

Perfect manifold 2D code under toroidal reference grid 3×2

n/n	Binary 2D code			
	Vector	Digit weights		
		(0,1)	(1,0)	(2,0)
1	(0,0)	0	1	1
2	(0,1)	1	0	0
3	(1,0)	1	0	0
4	(1,1)	1	1	0
5	(2,0)	0	0	1
6	(2,1)	1	0	1

Here (0,0) ≡ (1,0)+ (2,0) (mod 3, mod 2), (1,1) ≡ (0,1)+ (1,0) (mod 3, mod 2), (2,1) ≡ (0,1)+ (2,0) (mod 3, mod 2).

The perfect manifold 2D code takes information parameters S=7, R=1, n=3, t=2, m_1 =3, m_2 =2, P=6, where S – order of rotational symmetry, n-number of digits, t – dimensionality, 3×2 – sizes, and P- capacity of the perfect code.

The next is an example of perfect manifold 3D code with information parameters S=31, R=1, n=6, t=3, m_1 =2, m_2 =3, m_3 =5, P=30 (Table 3).

Table 3

Perfect manifold 3D code based on the IRB {(0,2,3), (1,1,2), (0,2,2), (1,0,3), (1,1,1),(0,1,0)}with information parameters S=31, R=1, n=6, t=3, m_1=2, m_2=3, m_3=5, P=30

n/n	Vector	Digit Weights					
		(0,2,3)	(1,1,2)	(0,2,2)	(1,0,3)	(1,1,1)	(0,1,0)
1	(0,0,0)	1	1	1	1	0	1
2	(0,0,1)	0	0	1	1	1	0
3	(0,0,2)	0	1	1	1	0	0
4	(0,0,3)	1	0	0	0	0	1
5	(0,0,4)	1	0	1	1	1	1
6	(0,1,0)	0	0	0	0	0	1
7	(0,1,1)	0	0	1	1	1	1
8	(0,1,2)	1	0	0	1	1	1
9	(0,1,3)	1	1	1	0	1	1
10	(0,1,4)	0	0	0	1	1	0
: :	: :	: :	: :	: :	: :	: :	: :
29	(1,2,3)	0	1	1	1	1	1
30	(1,2,4)	1	1	0	1	1	1

To see Tables 2 and 3, we observe all code combinations containing in the tables are "monolithic", namely consisting only consecutive symbols both "1" and "0" in the combination, as dividing a "whole" at two parts approved by the underlying "perfect" *S*–fold rotational symmetry-asymmetry ensemble. This remarkable property of the perfect manifold codes make it possible high performance vector data coding with elevated transmission speed. The 3D optimum monolithic code {(0,2,3), (1,1,2), (0,2,2), (1,0,3), (1,1,1), (0,1,0)} based on the 31-fold (S=31) "perfect" rotational symmetry and asymmetry ensembles forms complete set of P= 30 code combinations over 3D ignorable array 2×3×5 covered three-dimensional surface of a manifold exactly once (R=1). Code size of the coding system of six (n=6) binary digits coincides in number of cells in the array $m_1 \times m_2 \times m_3 = 2\times3\times5$.

Fig. 9. illustrates graphic representation a set of elegant ensembles "Gloria to Ukraine Stars" (GUS)[7].

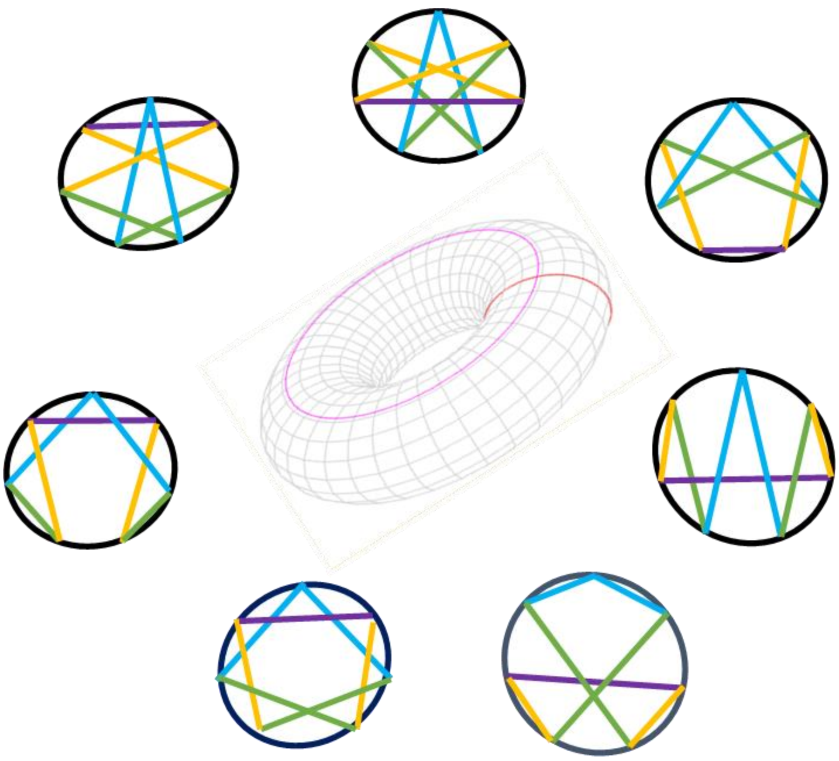

Fig. 9. Graphic representation a set of 8 elegant star's ensembles GUS

One of them depicted below (Fig. 10).

[7] Riznyk, V. V. (2019). *Combinatorial optimization of multidimensional systems. Models of multidimensional intelligence systems*. Monograph. Lviv: Lviv Polytechnic Publishing House, 168 p.

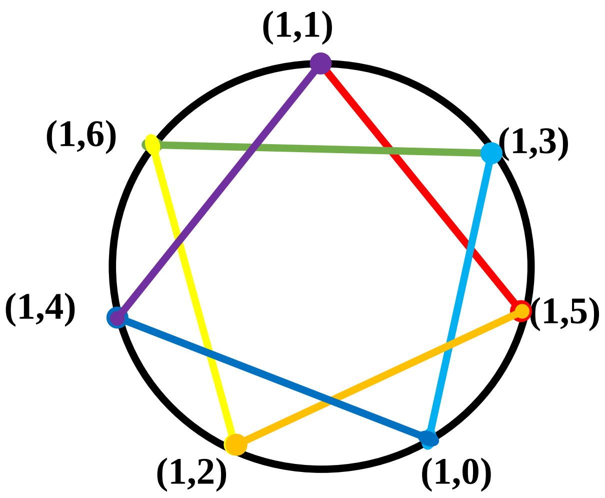

Fig. 10. Graphic representation one of a seven-pointed double GUSs {(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)}

The 2D vectors placed in vertexes of the graph allows any of two ways cyclic go-round in tracing path- ring {(1,1), (1,3), (1,5), (1,0), (1,2), (1,4), (1,6)} (black line), and star {(1,1), (1,5), (1,2), (1,6), (1,3), (1,0), (1,4)} (color lines), maintaining the remarkable properties of the perfect coding 2D vector data arrays. Specified graphs are useful in the visualizing information on the remarkable properties of the GUS manifolds.

Information embedded in elegant GUS ensembles provides an ability to configure perfect manifold coordinate systems to increase combinatorial varieties in the system extending the functionality of the processed vector data.

The vector information technologies under perfect manifold coordinate systems include the intelligent GUS combinatorial configurations, which allows a better understanding of the role of spatial symmetry-asymmetry in modern science and systems engineering.

CONCLUSIONS

Application of perfect manifold coordinate systems for information technologies provide new conceptual techniques for improving the quality indices of the technologies and management systems with respect to transmission content, and compression of vector data, transmission speed, and embody reliability of vector data coding, using design based on the remarkable properties of rotational symmetry and complementary asymmetry “elegant” relationship. The essence of the technology is processing vector information in the database of manifold coordinate system, where the basis is a set of coordinates smaller than the total number of coordinates of this coordinate system, which generates it by adding the latter.

Topological model of the perfect coordinate systems regarded as both algebraic groups in extensions of Galois fields, and intelligent non-redundant combinatorial configurations, generated from “elegant” ensembles of rotational symmetry composed from complementary asymmetries, namely the t-dimensional Ideal Ring Bundles (IRBs). Moreover, the optimization has been embedded in the underlying models. These design techniques make it possible to organize high performance vector information technology indexing “category – attribute” sets according to numbering node points set of perfect manifold coordinate grid for vector data processing under the minimized basis of the coordinate system. The remarkable properties and structural perfection of manifold coordinate systems, constructed on IRB and GUS configurations provide an ability processing big vector data to update completed tables with indexing of names, packages, procedures, etc. in the selected coordinate database, followed by processing in the database network. The underlying property makes optimum monolithic vector codes useful in applications to high performance vector data processing with respect to self-correcting, transmission speed, data reconstruction, and security. Application of the underlying relationships between the information parameters of manifold encoding systems and theoretical limitations for optimal solution of specific problems under manifold coordinate systems will make it possible to reach the best trade-off between performance and complexity for optimum processing information. These information technologies under manifold coordinate systems allows configure optimum two- and multidimensional vector data processing system, using innovative methods offering ample scope for progress in systems engineering, cybernetics, and industrial informatics.

SUMMARY

Processing large information content actualizes the problem of database shortening without loss of information. The essence of the proposed technology is processing vector data in the database of manifold coordinate system, where the basis is a set of coordinates smaller than the total their number. Binary code with vector weight discharges of the database allows indexing all values of vector data sets “category-attribute” according one-to-one set of manifold coordinate system grid. A code number of an attribute applied to each coordinate axis, as well as, to appropriate categories of the attributes. Each indexed vector data “category-attribute” mutually uniquely corresponds to the point with the eponymous set of the coordinate system. Besides, a combination of binary code with vector weight bits of the database is allowed, and the set of all

values of indexed vector data sets are the same that a set of numerical values. Perfect manifold coordinate system provides coding and processing big vector data with some changes of “category-attribute” sets at the same time. This information technology brought out relationship between the “elegant” ensembles of rotational symmetry and intelligent models of t-dimensional coordinate systems.

– Theoretically, infinitely many bases generate numerous varieties of sets of perfect coordinate systems of multidimensional spaces.

– The prospects for the development of vector information technologies are opened on the minimizing the basic structure of multidimensional information flow processing systems and the functionality of vector computer systems is expanded.

– There are many opportunities to apply manifold vector information technologies to numerous branches of sciences, education and advanced systems engineering.

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