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**METHOD OF TEACHING CONTINUOUS MAPPING  
OF TOPOLOGICAL SPACE AND ITS APPLICATION  
BY THE CONCEPT OF SOCIETY AND EDUCATION**

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In this paper, we will show how to teach general topology by using traditional methods with examples and show some applications of the rules of topology, especially in the concept of continuity between topological spaces. Application will be given as definition of society and education through continuity of topological spaces.

**Definition 1.** Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  are two topological spaces. A mapping  $f$  from  $X$  to  $Y$  is called continuous, if  $f^{-1}(U) \in \tau_1$  for every  $U \in \tau_2$ , i.e. if preimage of an every open subset of the space  $Y$  is an open subset of the space  $X$ . We denote continuous mapping  $f$  of the space  $X$  to  $Y$  in the form  $f: X \rightarrow Y$ .

**Example 1.**

1. Let  $f: (X, \tau_D) \rightarrow (Y, \tau)^-$  continuous mapping, as  $\forall V \in \tau$  follows  $f^{-1}(V) \in X \Rightarrow f^{-1}(V) \in \tau_D$ , where  $\tau_D^-$  discrete topology in  $X$  and  $\tau^-$  any topology in  $Y$ .

2. Let  $f: (X, \tau) \rightarrow (Y, \tau_A)^-$  continuous mapping, as  $\forall V \in \tau_A$  follows  $V = Y$  or  $V = \emptyset$  Hence  $f^{-1}(Y) = X \in \tau$  or  $f^{-1}(\emptyset) = \emptyset \in \tau$ , where  $\tau_A^-$  indiscrete topology in  $Y$  and  $\tau$  is an any topology in  $X$ .

3. Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$  and  $Y = \{x, y\}$ ,  $\tau_2 = \{\emptyset, Y, \{x\}\}$ . Let  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  and  $f(a) = f(b) = x, f(c) = y$ .

Then  $f^{-1}$  is continuous, as  $Y \in \tau_2$  consequently  $f^{-1}(Y) = X \in \tau$ ,  $\emptyset \in \tau_2$ ,  $f^{-1}(\emptyset) = \emptyset \in \tau$ ,  $\{x\} \in \tau_2$ ,  $f^{-1}(\{x\}) = \{a, b\} \in \tau$ .

**Example 2.** Prove, that a mapping  $f: R \rightarrow R$ , definition of  $\varepsilon - \delta$  continuity is followed from the definition of an open mapping.

Let  $f: R \rightarrow R$  is continuous in the point  $x \in R$ , if for every  $\varepsilon > 0$ , there exists  $\delta > 0$  such that,  $|f(y) - f(x)| < \varepsilon$  for every  $y$  where  $|y - x| < \delta$ . Then  $f$  is continuous, if it is continuous in every point  $p \in R$ .

*Proof.* Suppose that  $f: R \rightarrow R$  is continuous by definition  $\varepsilon - \delta$ . We will show that from the property (4) in preposition [1]  $f$  is continuous for open subsets. Let consider any point  $x \in R$  and any neighborhood  $V$  that contains  $f(x)$ . There exists basis element  $(c, d)$  contains  $f(x)$ , such that  $(c, d) \subset V$ . Let  $\varepsilon = \min(f(x) - c, f(x) + c)$ , notice that  $\varepsilon > 0$  as  $c < f(x) < d$ . It is trivial to  $(f(x) - \varepsilon, f(x) + \varepsilon) \subset (c, d) \subset V$  and contains  $x$ . Since  $f$  is continuous in the point  $x$ , there exists  $\delta > 0$  such that  $|y - x| < \delta$  sequentially  $|f(y) - f(x)| < \varepsilon$  for every  $y$ . Let  $U = (x - \delta, x + \delta)$ , it is clear, that it is the neighborhood of the point  $x$ . Let consider any point  $z \in f(U)$  such that  $z = f(y)$  for some  $y \in U$ . Let then we have  $x - \delta < y < x + \delta$  such that  $0 < y - x < \delta$ , therefore it is followed by  $|y - x| < \delta$ . We know that  $|z - f(x)| = |f(y) - f(x)| < \varepsilon$  such that  $f$  is continuous. Hence  $-\varepsilon < z - f(x) < \varepsilon$  such that  $f(x) - \varepsilon < z < f(x) + \varepsilon$ , and consequently  $z \in V$  as  $(f(x) - \varepsilon, f(x) + \varepsilon) \subset V$ .  $z \in f(U)$  is an arbitrary, and this shows that  $f(U) \subset V$ , and from the property (4) is fulfilled for  $f$  as  $x$  was arbitrary.

**Definition 2.** Continuous mapping  $f: X \rightarrow Y$  is called homeomorphism, if  $f$  bijective maps the space  $X$  onto  $Y$  and inverse image  $f^{-1}$  from  $Y$  to  $X$  is continuous.

Two topological spaces  $X$  and  $Y$  are called homeomorphic or topological, if there exists homeomorphism of the space  $X$  onto the space  $Y$  and denoted by  $X \cong Y$ .

**Example 3.** Let denote by  $X$  and  $X^*$  singleton sets with two topologies  $\tau$  and  $\tau^*$ , respectively. Let  $i: \tau \rightarrow \tau^*$  is an identity mapping.

1. We will show that  $i$  is continuous, if only if, when  $\tau^*$  is weaker than  $\tau$ .
2. We will show that  $i$  is a homeomorphism, if only if, when, when  $\tau^* = \tau$ .

Solution: 1) Notice that, inverse of an identity mapping is a domain, thus it is an inverse for any subset  $A \subset X = X^*$ , so we have  $i(A) = i^{-1}(A) = A$ .

( $\Rightarrow$ ) Suppose that  $i$  is continuous and consider any open subset  $U \in \tau$ . Then we have  $i^{-1}(U) = U$  open subset in  $\tau^*$  as  $i$  is continuous. Since  $U$  is an arbitrary, it shows that,  $\tau \subset \tau^*$  consequently  $\tau^*$  is weaker.

( $\Leftarrow$ ) Let consider that  $\tau^*$  is weaker, as  $\tau \subset \tau^*$ . Consider any open set  $U \in \tau$  such that  $U \in \tau^*$ , i.e.  $U$  is also open in  $\tau^*$ . Since  $i^{-1}(U) = U$ , this shows that  $i$  is continuous by the definition of continuity.

2) It is obvious, that  $i$  is bijective as domain and image are same i.e.  $i^{-1} = i$ .

$i$  is a homeomorphism, if only if, when  $i$  and  $i^{-1}$  are continuous. Ssequently  $\tau^*$  is weaker than  $\tau$ , and  $\tau$  is weaker than  $\tau^*$  ( from (1)), hence  $\tau^* \subset \tau$  consequently  $\tau^* = \tau$

In topology, elements function in multiple surfaces and are affected by each surface. In this way, topology changes our understanding of “familiar social science objects of research by mapping out how such objects change and how they relate, in this process, to other changing objects in multiple, relational spaces”. Multiplicity means that elements belong to more than one surface. An element that is part of two topologies is the site of three effects, the effects of each surface and the effects of the surfaces formed by

the combination of both surfaces. This does not mean that all the topologies active at some point are equally affective at all times. While elements of scientific management and education for efficiency may still resonate, there have been certain spaces and time where these elements were more affective. Another important attribute is that topologies continuously, though not always rapidly, change their internal dynamics, or deform. In our increasingly topological society, movement – as the ordering of continuity – composes the forms of social and cultural life themselves. This is not a matter of one rationality displacing the other, but of their overlapping and mutual implication such that the continuity of movement – or the continuum – becomes fundamental to contemporary culture. This continuous multiple deformation occurs across axes that are immanent to a topology and so deformation occurs along certain ‘lines’ or according to certain principles. The famous deformation of a coffee cup into a doughnut is possible because it follows the line of ‘having only one hole’. But topology also theorizes fuzzier, yet mathematically rigorous, ‘shape consistency’ under deformation. This can usefully be compared to other things that change yet are held to remain the same, such as a family or community or group – vitalities, that is, intangible-but-real-entities that remain despite turnover in membership.

In topological thinking, unless one of the co-constitutive surfaces is ruptured, and its topological relation is lost, all topologies continue to produce effects. So, unlike the virtual, which is beyond experience and the experienced actual, topological figures cut across the distinction of the virtual and actual. The movement, the process at stake is not the generation of an actual by a virtual, but the deformation of, as it were, two actuals into one another via their topological properties. Topological surfaces enfold and re-enfold each other in a complicated dance of continuously deforming multiplicity. The main questions concern the topological principles at work on an element and the ways they are inflected and deflected by other topological principles at work on and through that element. The surface formed by, the interaction between two or more topologies is an effect of the multiple binding principles at work in the various surfaces of which an element is a part. The immanence of the principles at work in every topological surface, though, means that the effects of the deformations of an element belonging to multiple surfaces are not found. ‘Interior’ and ‘exterior’ make a different sense in topological thinking, as insides and outsides are continuous ... borders of inclusion and exclusion do not coincide with the edges of a demarcated territory, and it is the mutable quality of relations that determines distance and proximity, rather than a

singular and absolute measure. In a topological society, the nation is not necessarily bigger or stronger than, say, an electricity meter, and the domestic is not necessarily situated at a lower level than a map of the world.

The education topological function at work in the assemblage is made up of multiple constant connections of topologies; architectural topologies, individualizing surfaces, amassing topologies, knowledge topologies, semiotic surfaces, corporeal topologies, subjectivizing topologies – recalling that every element or point exists in multiple topologies at multiple times. The ‘teacher’, for example, is actualized through multiple topological affects (including compartments, materials, curricula), as it is constructed by topologies that connect and by the ways those connections work; “form relate to populations, populations imply codes, and codes fundamentally include phenomena of relative decoding that are all the more usable, composable, and addable by virtue of being relative, always beside. Co-constitutive topologies construct this subject ‘teacher’. Space is multi-dimensional and constituted through mutually implicated; so, it is not hard to see the teaching as produced through these topological effects in the same space at the same time. It is this co-constitution through multiple topologies that is crucial because they are coterminous, overlapping, connecting and continuously deforming. They are machinic, in being “simultaneously located at the intersection of the contents and expression on each stratum, and at the intersection of all of the strata with the plane of consistency. They rotate in all directions, like beacons”. Rotation, stretching, deforming, education is a dynamic topological affect at the molecular level. This requires addressing bodies, enunciations and their relations; for example, mapping is the “intermingling of bodies defining feudalism: the body of the earth and the social body; the body of the overlord, vassal, and serf; the body of the knight and the horse and their new relation to the stirrup; the weapons and tools assuring a symbiosis of bodies” as well as those “statements, expressions, the juridical regime of heraldry, all of the incorporeal transformations, in particular, oaths and their variables: the collective assemblage of enunciation”. Within each topological assemblage, what is important is the ways that lines of deterritorialization form points of intersection between enunciative acts on the one hand, and the machinic assemblage of bodies, their attributes, actions and capacities on the other.

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